Gravitational correction and Weak gravity conjecture

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ABSTRACT: We consider the gravitational correction to the running of gauge coupling. Weak gravity conjecture implies that the gauge theories break down when the gravitational correction becomes greater than the contribution from gauge theories. This observation can be generalized to non-Abelian gauge theories in diverse dimensions and the cases with large extra dimensions.

In [1], Vafa suggested that gravity and the other gauge forces can not be treated independently and the vast series of semi-classically consistent effective field theories which belong to swampland are actually inconsistent after gravity is included. The authors in [2] proposed the weak gravity conjecture which can be most simply stated as gravity is the weakest force. This conjecture implies that in a four-dimensional theory with gravity and a U(1) gauge theory, there is a new intrinsic UV cutoff for the U(1) gauge theory

$$\Lambda \sim g M_4, \tag{1}$$

where $M_4 = 1.4 \times 10^{19}$ GeV is the four-dimensional Planck scale and g is the U(1) gauge coupling.

In [3], some heuristic arguments on the weak gravity conjecture in the asymptotical dS and AdS space implies that a lower bound on the U(1) gauge coupling g, or equivalently, the absolute value of the cosmological constant gets an upper bound

$$|\rho_V| \leq g^2 M_4^4, \tag{2}$$

in order that the U(1) gauge theory can survive in four dimensions. This result has a simple explanation in string theory, i.e. the string scale $\sqrt{\alpha'}$ should not be greater than the size of the cosmic horizon. On the other hand, the most exciting possibility raised by large extra dimensions [4–9] is that the fundamental Planck scale may be much lower than the apparent four-dimensional Planck scale. This implies that we may begin to experimentally access the dynamics of quantum gravity sooner than previously anticipated. In [10], we proposed that the intrinsic UV cutoff for U(1) gauge theory with large extra dimensions is proportional to the fundamental Planck scale, not the four-dimensional Planck scale. This new energy scale predicted by weak gravity conjecture may be relevant to the physics at LHC. In [11], Banks et al. generalized the weak gravity conjecture to higher dimensions. Some other related topics are discussed in [12–19].

In this paper, we investigate the gravitational correction to the Callan-Symanzik β function in diverse dimensions and the case with large extra dimensions, and provide a new viewpoint on the weak gravity conjecture. We do not take into account three or lower dimensions as gravity does not contain propagating degree of freedom in these dimensions, even though some of our results may be applicable to three dimensional cases as well.

Robinson and Wilczek in [20] consider the one-loop gravitational correction to the running of gauge theory couplings in four dimensions. They take into account the Feynman diagrams involving a vertex dressed by graviton exchange. The gluon-graviton vertex in four dimensions is proportional to Λ/M_4, where Λ is the energy scale. After considering the graviton exchange, one-loop β function for the running of gauge coupling takes the form

$$\beta \equiv \frac{dg}{d\ln\Lambda} = -\frac{b_0}{(4\pi)^2}g^3 - a_0\left(\frac{\Lambda}{M_4}\right)^2 g. \tag{3}$$

The first term on the right hand side of (3) takes the form of familiar non-gravitational contribution and the second term includes the gravitational contribution. Because gravitons don't carry any gauge charges, the value of b_0 is determined by the matter content and the

gauge group. The unknown coefficient a_0 in eq. (3) is determined to be $a_0 = 3/\pi$ in [20]. When gravitational effects become significant, the local effective gauge theory is broken down. Requiring that the gravitational correction is not greater than the contribution from gauge theory yields

$$\Lambda \leq CgM_4, \tag{4}$$

where

$$C = \frac{1}{4\pi}\sqrt{\frac{|b_0|}{a_0}}. \tag{5}$$

This result is just the same as [2]. Now weak gravity conjecture is interpreted as the condition for that the gravitational correction is smaller than the contribution from gauge fields. The above argument can be easily generalized to non-Abelian gauge field theories. For example, $b_0 = \frac{11}{3}N$ for pure SU(N) Yang-Mills theory. The weak gravity conjecture predicts

$$\Lambda \leq C_N\sqrt{Ng^2}M_4, \tag{6}$$

where $C_N \simeq 0.16$. The combination with Ng^2 is nothing but the 't Hooft coupling. It is just what we expect.

Here a suggestive coefficient C_1 is obtained. In standard model, $b_0 = -41/6$ for $U(1)_Y$ and thus $C = 0.21$. For $SU(2)_L$, $b_0 = 19/6$ and $C = 0.14$; for $SU(3)_C$, $b_0 = 7$ and $C = 0.22$. However, it is well known that the pure U(1) electromagnetism and $\mathcal{N} = 4$ super-Yang-Mills in four dimensions are scale free theories with $b_0 = 0$. Weak gravity conjecture implies these two theories cannot self-consistently survive with gravity.

In the case with large extra dimensions, the fundamental Planck scale can be much lower than the apparent four-dimensional Planck scale. If the fundamental Planck scale is roughly $1\,\text{TeV}$, it will open a window to detect quantum gravity in LHC in the near future. Since the fundamental Planck is quite low, we expect that gravity provides a significant correction on the one-loop β function. In this scenario, gravitons propagate in the whole spacetime, but the gauge fields only propagate in four dimensions. The interaction between graviton and gauge fields is described by the action

$$S = \frac{1}{2\kappa_d^2}\int d^d x \sqrt{-det(g_{mn})}R + \int d^4 x \sqrt{-det(g_{\mu\nu})}\frac{1}{4}\text{Tr}(F^2), \tag{7}$$

where $\kappa_d^2 \sim G_d$ is the Newton coupling constant in d dimensions. Consider the quantum fluctuation of the gravitational degrees of freedom around the Minkowski metric as

$$g_{mn} = \eta_{mn} + \kappa_d h_{mn}. \tag{8}$$

The action (7) becomes

$$S \sim \int d^d x (\partial h)^2 + \int d^4 x \kappa_d \eta^{\mu\rho}\partial_\mu A_\nu \partial_\rho A_\sigma h^{\nu\sigma} + \cdots. \tag{9}$$

The interaction term between graviton and gauge field is proportional to positive powers of κ_d. Dimensional analysis implies that the gluon-graviton vertex is proportional to $\kappa_d \Lambda^{\frac{d-2}{2}}$.

If we take the low energy limit, all interaction terms between gauge field and graviton drop out. The running of gauge coupling is governed by a modified β function which is given by

$$\beta = -\frac{b_0}{(4\pi)^2}g^3 - c_0\kappa_d^2\Lambda^{d-2}g = -\frac{b_0}{(4\pi)^2}g^3 - c_0\left(\frac{\Lambda}{M_d}\right)^{d-2}g, \tag{10}$$

where $M_d = G_d^{-\frac{1}{d-2}}$ is the d-dimensional Planck scale. Demanding that the gravitational correction is less than the gauge contribution leads to

$$\Lambda \leq C_E g^{\frac{2}{d-2}}M_d, \tag{11}$$

with

$$C_E = \left(\frac{1}{16\pi^2}\left|\frac{b_0}{c_0}\right|\right)^{\frac{1}{d-2}}. \tag{12}$$

eq. (11) is the same as the result in [10]. Similarly, for the SU(N) gauge theory, $b_0 \sim N$ and then $\Lambda \leq (Ng^2)^{\frac{1}{d-2}}M_d$. In [21], the authors naively use the results in [20] for the case with large extra dimensions, which is not reliable.

In the scenario with large extra dimensions, not only the black holes are possibly produced [22, 23], but also a new intrinsic UV cutoff for gauge theories in standard model emerges. We can also detect the gravitational effects according to the modification for the running of the gauge coupling. The value of unknown coefficient c_0 will be determined in the future.

Our previous arguments on the gauge theories coupled to gravity can be generalized to the cases in higher dimensions. In d dimensions, gauge coupling has dimensions of $[mass]^{2-d/2}$. For $d > 4$, gauge coupling takes dimensions of positive power of length and gauge interaction becomes irrelevant. The gauge coupling depends on the energy scale Λ according to

$$\frac{dg_d}{d\ln\Lambda} = -f_0\Lambda^{d-4}g_d^3 - h_0 G_d\Lambda^{d-2}g_d, \tag{13}$$

where g_d is d-dimensional gauge coupling, f_0 and h_0 are the numerical coefficients. The factors of Λ in (13) have been inserted by dimensional analysis. Again requiring that the gravitational correction is less than the contribution from gauge fields yields

$$\Lambda \leq g_d M_d^{\frac{d-2}{2}}, \tag{14}$$

which is just the same as that in [11]. For SU(N) gauge theories, we just need to replace g with $\sqrt{N}g$.

We are also interested in investigating the weak gravity conjecture in the asymptotical de Sitter and anti-de Sitter space. Following the idea of [3], these effective field theories breaks down when the curvature radius of the background is less than the shortest reliable distance for the field theories $1/\Lambda$. The curvature radius of the background is roughly $L \sim 1/\sqrt{G_d|\rho_V|}$, where ρ_V is the cosmological constant. Requiring $L > 1/\Lambda$ yields an upper bound on the cosmological constant ρ_V in d dimensions

$$|\rho_V| \leq Ng_d^2 M_d^{2(d-2)} \tag{15}$$

for SU(N) gauge theory. For U(1) gauge theory in four dimensions, the result is just the same as that in [3].

Consider the brane world scenario on Dp-brane. Now $d = p + 1$. The gauge coupling for U(1) gauge theory on a Dp-brane is

$$g_d \sim g_s^{\frac{1}{2}} M_s^{-\frac{p-3}{2}}. \tag{16}$$

Taking toroidal compactification, the d-dimensional Planck scale is related to the size of the extra dimensions R by

$$M_d^{p-1} \sim g_s^{-2} M_s^{p-1} (M_s R)^{9-p}. \tag{17}$$

The tension of Dp-brane

$$T_p \sim \frac{M_s^{p+1}}{g_s} \tag{18}$$

provides an effective cosmological constant on the brane. Eq. (15) is translated into

$$g_s \leq (M_s R)^{(9-p)}. \tag{19}$$

On the other hand, the Hubble constant on the brane takes the form

$$H^2 \sim \frac{T_p}{M_d^{p-1}} \sim \frac{g_s M_s^2}{(M_s R)^{9-p}}. \tag{20}$$

With the viewpoint of string theory, requiring that the string length should be shorter than the curvature radius of the background, $l_s \leq H^{-1}$, yields $g_s \leq (M_s R)^{(9-p)}$ which is the same as (19); otherwise, string can be not taken as a point-like particle and the full string theory should be involved.

Similarly, for a stack of N Dp-branes, the gauge group becomes SU(N) for the open string modes on these D-branes and the effective cosmological constant is given by $\rho_V \sim N T_p$. Now eq. (15) becomes

$$g_s \leq (M_s R)^{9-p}. \tag{21}$$

On the other hand, the Hubble constant on these branes takes the form

$$H^2 \sim \frac{N T_p}{M_d^{p-1}} \sim \frac{N g_s M_s^2}{(M_s R)^{9-p}}. \tag{22}$$

Requiring $l_s \leq H^{-1}$ leads to

$$g_s N \leq (M_s R)^{(9-p)}, \tag{23}$$

which is more stringent than (21). It seems that our constraint on the SU(N) gauge theory in the asymptotical de Sitter space is quite loose.

In this paper, we provide a new observation on the weak gravity conjecture which is independent on the arguments based on black hole [2, 11]. The gravitational correction should be less than the contribution from the gauge theories; otherwise, the effective description of the gauge theories breaks down. The straightforward framework for quantum

gravity, general relativity quantized for small fluctuations around flat space, is nonrenormalizable quantum field theory. In [20], the authors used the background field method to calculate the gravitational correction to the β function for the gauge theories. Their results should be reproduced in a reliable fundamental theory of quantum gravity. Our investigation provides the new insight into the weak gravity conjecture at the quantum level and can be generalized to discuss the weak gravity conjecture for the scalar field theories [24].

Acknowledgments

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